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Applied Thermal Engineering 20 (2000) 831–841

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APPLIED THERMAL  
ENGINEERING

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# A correlation of optimal heat rejection pressures in transcritical carbon dioxide cycles

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Received 10 April 1999; accepted 25 July 1999

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## Abstract

In this work, a cycle simulation model has been developed to optimize the coefficient of performance (COP) of transcritical carbon dioxide air-conditioning cycles. The analysis shows that the COP of the transcritical carbon dioxide cycle varies nonmonotonically with the heat rejection pressure; a maximum COP occurs at an optimal heat rejection pressure. It is further revealed that the values of the optimal heat rejection pressure mainly depend on the outlet temperature of the gas cooler, the evaporation temperature, and the performance of the compressor. Based on the cycle simulations, correlations of the optimal heat rejection pressure in terms of appropriate parameters are obtained for specific conditions. The results are of significance for the design and control of the transcritical carbon dioxide air-conditioning and heat pump systems © 2000 Elsevier Science Ltd. All rights reserved.

*Keywords:* Carbon dioxide; Refrigeration; Transcritical cycle

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## 1. Introduction

The history of the use of carbon dioxide as a refrigerant dates back to the end of the last century [1]. The American Alexander Twining proposed to use carbon dioxide as a refrigerant for vapor compression systems in his patent in 1850. The first carbon dioxide system (ice

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### Nomenclature

$C$	empirical constant in Eq. (6)
COP	coefficient of performance
$h$	specific enthalpy (kJ/kg)
$K$	empirical constant in Eq. (6)
$p$	pressure (bar)
$q$	specific refrigerating effect (kJ/kg)
$t$	temperature (°C)
$w$	specific compression work (kJ/kg)

### Greek symbol

$\eta$	efficiency of compressor
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### Subscripts

c	gas cooler or heat rejection
e	evaporator
is	isentropic
opt	optimal value
sh	superheat
t	theoretical

machine) was built by Thaddeus S. C. Lowe in about 1869 in Jackson, Mississippi. The design of a carbon dioxide compressor by a German engineer, Franz Windhausen, in 1886 brought carbon dioxide systems into wide use. Subsequently, carbon dioxide as a refrigerant was widely used not only in marine systems but also in general refrigeration applications. Starting from the 1940s, carbon dioxide as a refrigerant was displaced by chlorofluorocarbons (CFCs), the so-called “safety refrigerants”.

In 1974, two American scientists published their famous ozone depletion hypothesis in which they claimed that CFCs would destroy the ozone layer of the Earth’s atmosphere [2]. This discovery has since resulted in an increasingly hectic search for CFC substitutes. With zero ozone depleting potentials (ODPs), several hydrofluorocarbons (HFCs) and HFC blends seem to be possible CFC and hydrochlorofluorocarbon (HCFC) substitutes. However, HFCs have relatively high global warming potentials (GWPs), and these substances are synthetic compounds foreign to nature. Extensive use of these artificial substances may cause other environmental problems for mankind. For this reason, increased attention has been paid in recent years to the use of natural substances such as carbon dioxide, ammonia, hydrocarbons, water and air as refrigerants.

As suggested by Lorentzen and Pettersen [3], the use of carbon dioxide as a refrigerant may provide a totally safe, economical and cost-effective “natural” solution in many applications. As a refrigerant, carbon dioxide has a zero ODP because it can be recovered from industrial waste gases [3]. Various research activities on the use of carbon dioxide as refrigerant are

currently being carried out throughout the world. These applications include automobile air-conditioning [3], railway refrigeration and air-conditioning [4], heat pumps [5], residential air-conditioning [6] and secondary-loop systems [7].

Due to the fact that the critical temperature of carbon dioxide (31.1°C) is usually lower than typical values of the heat rejection temperature of air-conditioning and heat pump systems, the refrigeration systems using carbon dioxide as a refrigerant have to undergo a so-called transcritical cycle, in which the heat rejection process takes place above the supercritical pressure while the heat absorbing process occurs under the subcritical condition. Therefore, the most notable feature of the transcritical cycle is that the condensing process in the conventional subcritical cycle is replaced by a cooling process in a so-called gas cooler. It is well known that for the conventional subcritical system the coefficient of performance (COP) increases with the decrease of the heat rejection (condensation) pressure. For the transcritical carbon dioxide cycle, however, the variation of the COP with the heat rejection pressure exhibits a nonmonotonic change due to the fact that the heat rejection temperature is independent of the heat rejection pressure in the supercritical region. Previous studies [8,9] show that there exists an optimal heat rejection (cooling) pressure that gives a maximum COP.

For the purpose of the optimization and control of a transcritical CO<sub>2</sub> system, it is of interest to obtain a correlation of the optimal heat rejection pressure in terms of appropriate parameters. For a theoretical thermodynamic cycle, Inokuty's graphical method can be used to find the optimal heat rejection pressure [8]. However, this method cannot be used to analyze more practical cases in which the optimal heat rejection pressure may be affected by the efficiency of the compressor. In this work, a thermodynamic model with the isentropic efficiency of a carbon dioxide compressor taken into consideration is developed. Based on this model, a steady-state simulation of transcritical CO<sub>2</sub> systems, using Engineering Equation Solver (EES) software [10] and the carbon dioxide property data in REFPROP [11], has been carried out. The analysis reveals that the values of the optimal heat rejection pressure mainly depend on the outlet temperature of the gas cooler, the evaporation temperature and the performance of the compressor. Based on the cycle simulations, a correlation of the optimal heat rejection pressure in terms of appropriate parameters is obtained.

## 2. Cycle analysis and simulation

Consider a simple transcritical carbon dioxide system including a compressor, a gas cooler, a throttling valve, and an evaporator. The corresponding  $p$ - $h$  (pressure-specific enthalpy) diagram, shown in Fig. 1, indicates that this transcritical CO<sub>2</sub> cycle consists of an isentropic compression process (1–2), an isobaric heat rejection process (2–3), an adiabatic expansion process (3–4), and an isobaric evaporation process (4–1). The theoretical coefficient of performance (COP<sub>t</sub>) of this cycle is defined as

$$\text{COP}_t = q/w = (h_1 - h_4)/(h_2 - h_1) = (h_1 - h_3)/(h_2 - h_1) \quad (1)$$

where  $q$  represents the specific refrigeration effect,  $w$  represents the specific compression work, and  $h_1$ ,  $h_2$ ,  $h_3$ , and  $h_4$  denote the specific enthalpies of CO<sub>2</sub> at the corresponding points shown in Fig. 1.

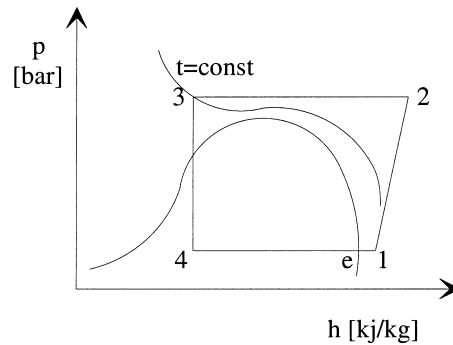


Fig. 1.  $p$ - $h$  diagram of a simple transcritical  $\text{CO}_2$  cycle.

We now also consider a revised transcritical  $\text{CO}_2$  system for automobile air-conditioners devised by Lorentzen and Pettersen [3]. As illustrated in Fig. 2, in this system, a receiver is positioned at the evaporator outlet and an internal heat exchanger (IHE) is used. Based on the schematic  $p$ - $h$  diagram of this system shown in Fig. 3, the theoretical  $\text{COP}_t$  of this cycle is defined as

$$\text{COP}_t = q/w = (h_e - h_b)/(h_2 - h_1) \quad (2)$$

where  $h_1$ ,  $h_2$ ,  $h_3$ ,  $h_4$ ,  $h_b$ ,  $h_e$  are the specific enthalpies of  $\text{CO}_2$  at the corresponding points in Figs. 2 and 3. Referring to Fig. 3 and assuming that the internal heat exchanger is perfectly insulated from its surroundings, we can readily show that

$$h_e - h_b = h_1 - h_3 \quad (3)$$

Substituting Eq. (3) into (2) yields

$$\text{COP}_t = (h_1 - h_3)/(h_2 - h_1) \quad (4)$$

A comparison between Eqs. (1) and (4) shows that the COP of the revised transcritical carbon dioxide cycle with an IHE (shown in Fig. 3) is the same as that of the simple cycle (shown in Fig. 1), if the thermodynamic parameters at the corresponding points 1, 2 and 3 are the same.

When the isentropic efficiency  $\eta_{is}$  of the compressor is taken into account, the COP of the

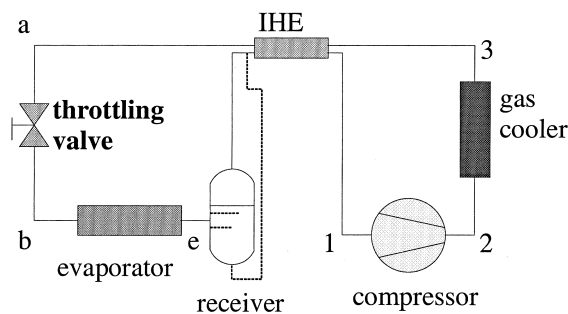


Fig. 2. Flow circuit of the transcritical  $\text{CO}_2$  cycle with IHE.

transcritical carbon dioxide system can be calculated based on

$$\text{COP} = \eta_{\text{is}} \text{COP}_t = \eta_{\text{is}}(h_1 - h_3)/(h_2 - h_1) \quad (5)$$

The isentropic efficiency  $\eta_{\text{is}}$  is mainly influenced by the ratio of the heat rejection pressure  $p_c$  to the evaporation pressure  $p_e$ . For small  $p_c/p_e$ ,  $\eta_{\text{is}}$  can be approximated by

$$\eta_{\text{is}} = C - K(p_c/p_e) = C[1 - (K/C)(p_c/p_e)] \quad (6)$$

where  $K$  and  $C$  are empirical constants. Combining Eqs. (5) and (6) and expressing the enthalpies in terms of the corresponding pressures and temperatures give

$$\text{COP} = C[1 - (K/C)(p_c/p_e)][h_1(p_e, t_{\text{sh}}) - h_3(p_c, t_c)]/[h_2(p_c, t_{\text{sh}}) - h_1(p_e, t_{\text{sh}})] \quad (7)$$

where  $t_{\text{sh}} = t_1 - t_e$  is the superheat. Eq. (7) indicates that the system COP depends on the heat rejection pressure  $p_c$ , the outlet temperature  $t_c$  of the gas cooler, the evaporation temperature  $t_e$ , the superheat  $t_{\text{sh}}$  as well as the two empirical constants  $K$  and  $C$  in Eq. (6), i.e.:

$$\text{COP} = \text{COP}(p_c, t_c, t_e, t_{\text{sh}}, K, C) \quad (8)$$

Using Eq. (7), the effect of various parameters on the COP of the transcritical carbon dioxide system can be investigated. The results will be reported in the next section.

Previous studies [8,9] show that there exists an optimal heat rejection pressure that gives a maximum COP for given values of the outlet temperature  $t_c$  of the gas cooler and the evaporator temperature  $t_e$ . At the optimal heat rejection pressure  $p_{\text{opt}}$ , the partial derivative of COP with respect to the heat rejection pressure  $p_c$  should equal zero:

$$\left[ \frac{\partial \text{COP}}{\partial p_c} \right]_{p_c=p_{\text{opt}}} = 0 \quad (9)$$

From Eqs. (7) and (9), it can be inferred that the empirical constants  $C$  and  $K$  in Eq. (6) exert their influence on the optimal heat rejection pressure  $p_{\text{opt}}$  in the form of  $K/C$ . Accordingly, we have

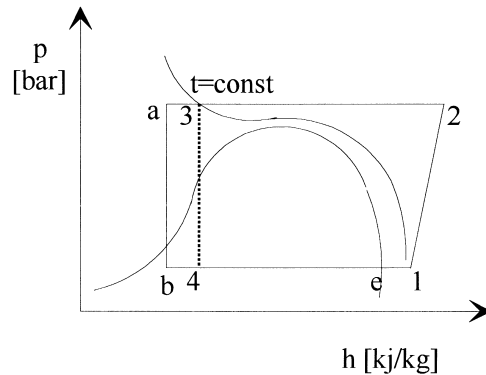


Fig. 3.  $p$ - $h$  diagram of the transcritical  $\text{CO}_2$  cycle with IHE.

$$p_{\text{opt}} = p_{\text{opt}}(t_c, t_e, t_{\text{sh}}, K/C) \quad (10)$$

Based on the above analysis, a steady state simulation program for transcritical CO<sub>2</sub> air-conditioning and heat pump systems using EES software [10] and REFPROP [11] CO<sub>2</sub> property data is developed. In this simulation program, a half-step searching method [12] is used to find the optimal value of the heat rejection pressure.

### 3. Results and discussion

The transcritical CO<sub>2</sub> cycles were simulated under a wide range of conditions such as  $-10^\circ\text{C} < t_e < 20^\circ\text{C}$ ,  $30^\circ\text{C} < t_c < 60^\circ\text{C}$ ,  $0^\circ\text{C} < t_{\text{sh}} < 20^\circ\text{C}$ ,  $71 \text{ bar} < p_c < 120 \text{ bar}$ , and  $0 < K/C < 0.3$ . In what follows, we shall first report the results of a parametric study in which the influence exerted by various cycle parameters is examined. We shall then present a correlation of the optimal heat rejection pressure in terms of appropriate cycle parameters.

#### 3.1. Parametric study

A parametric study was conducted using the following expression for the isentropic efficiency  $\eta_{\text{is}}$ :

$$\eta_{\text{is}} = 1.003 - 0.121(p_c/p_e) \quad (11)$$

which was obtained by best fitting the experimental data of a carbon dioxide compressor made by Danfoss A/S.

The influence of the heat rejection pressure  $p_c$  on the refrigeration effect  $q$ , the compression work  $w$ , and the COP for  $t_e = 10^\circ\text{C}$ ,  $t_c = 40^\circ\text{C}$  and  $t_{\text{sh}} = 5^\circ\text{C}$  is presented in Fig. 4. It is seen that the required compression work increases nearly linearly with increasing heat rejection pressure. It is also noted from Fig. 4 that as the heat rejection pressure is increased, the refrigeration

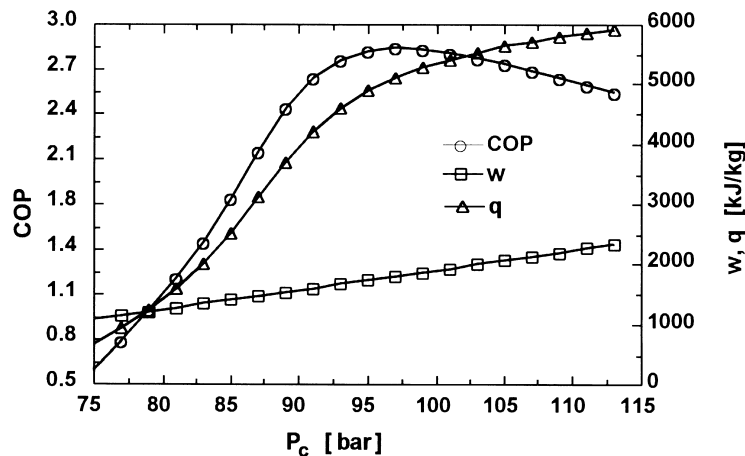


Fig. 4. The effect of heat rejection pressure  $p_c$  on the refrigerating effect, the compression work and the COP.

effect is increased rapidly for lower heat rejection pressures but increased relatively slowly when the heat rejection pressure reaches a certain value. Moreover, it is evident from Fig. 4 that there exists an optimal heat rejection pressure which gives a maximum COP. This result suggests that it is desirable that a transcritical CO<sub>2</sub> system should operate at or near its optimal heat rejection pressure in order to keep its maximum COP.

Fig. 5 shows the effect of the heat rejection pressure  $p_c$  for  $t_e=10^\circ\text{C}$  and  $t_{sh}=5^\circ\text{C}$  as the outlet temperatures of the gas cooler is varied from 32 to 36 and  $40^\circ\text{C}$ . It is seen that the optimal heat rejection pressure depends on the outlet temperature of the gas cooler, a higher outlet temperature of the gas cooler leading to a higher optimal heat rejection pressure.

The influence of the outlet temperature of the gas cooler on the COP for  $t_e=10^\circ\text{C}$  and  $t_{sh}=5^\circ\text{C}$  at various heat rejection pressures  $p_c=80, 90$  and  $100$  bar is presented in Fig. 6. The COP is seen to drop rapidly with the increase of the outlet temperature  $t_c$ . This is because with the increase of the outlet temperature of the gas cooler the refrigerating effect decreases, whereas the compression work remains the same.

Fig. 7 shows the effect of the evaporator temperature  $t_e$  on the COP for  $t_c=35^\circ\text{C}$  and  $t_{sh}=5^\circ\text{C}$  at various heat rejection pressures  $p_c=80, 90$  and  $100$  bar. The COP increases rapidly as the evaporator temperature  $t_e$  is increased. This behavior is similar to that observed in conventional subcritical compression refrigeration cycles.

Fig. 8 shows the effect of the superheat  $t_{sh}$  on the COP for  $t_c=35^\circ\text{C}$  and  $t_e=10^\circ\text{C}$  at various heat rejection pressures  $p_c=80, 90$  and  $100$  bar. The effect of the superheat  $t_{sh}$  is seen to be relatively unimportant as compared with the other parameters discussed in the preceding paragraphs.

### 3.2. Correlations of optimal heat rejection pressure

In the light of the fact that the influence exerted by the superheat  $t_{sh}$  on the COP is rather weak (see Fig. 8), we may neglect the effect of the superheat  $t_{sh}$  in determining the optimal heat rejection pressure. Accordingly, Eq. (10) becomes

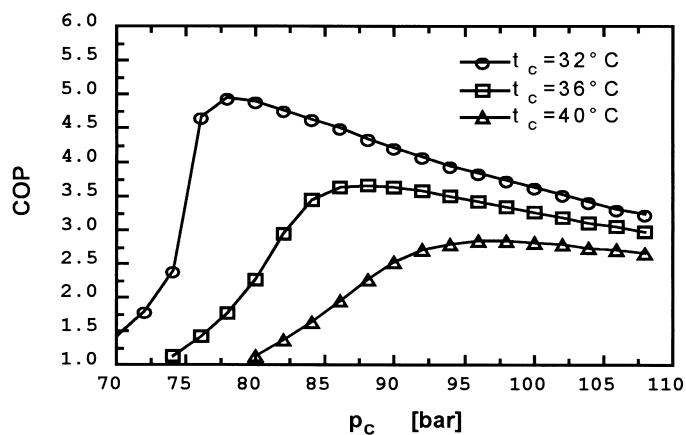


Fig. 5. The effect of the heat rejection pressure  $p_c$  on the COP at different outlet temperatures of the gas cooler.

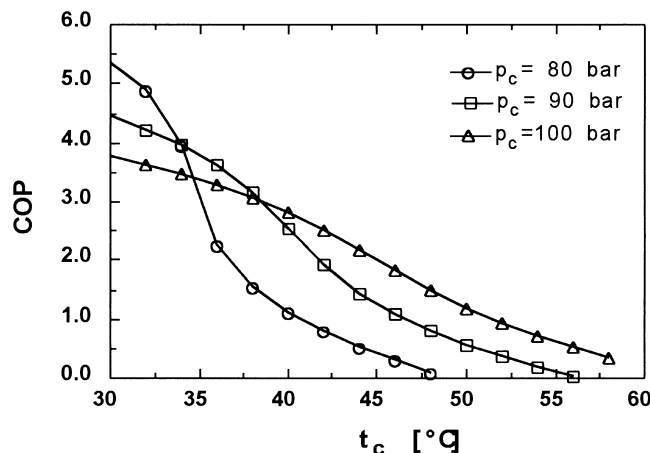


Fig. 6. The effect of the outlet temperature of the gas cooler  $t_c$  on the COP at different heat rejection pressures.

$$p_{\text{opt}} = p_{\text{opt}}(t_c, t_e, K/C) \quad (12)$$

Eq. (12) implies that the optimal heat rejection pressure for a transcritical  $\text{CO}_2$  cycle depends on three major parameters: the outlet temperature of the gas cooler, the evaporation temperature, and the performance of the compressor used in the system (represented by  $K/C$ ). To view the influence of the parameter  $K/C$  on the optimal heat rejection pressure  $p_{\text{opt}}$ , the results simulated based on Eq. (12) for  $t_e = 0^\circ\text{C}$  and  $K/C = 0, 0.1, 0.2,$  and  $0.3$  are presented in Fig. 9. As seen from Fig. 9, the optimal heat rejection pressure increases nearly linearly with increasing outlet temperature of the gas cooler when  $K/C \leq 0.2$ . For  $K/C > 0.2$ , however, the optimal heat rejection pressure  $p_{\text{opt}}$  increase rapidly for lower outlet temperatures of the gas cooler, whereas a more gradual increase is seen for higher outlet temperatures of the gas cooler. The following correlation was obtained based on a least square fit of the simulated results for the optimal heat rejection of a transcritical  $\text{CO}_2$  cycle for  $0 \leq K/C \leq 0.2$ :

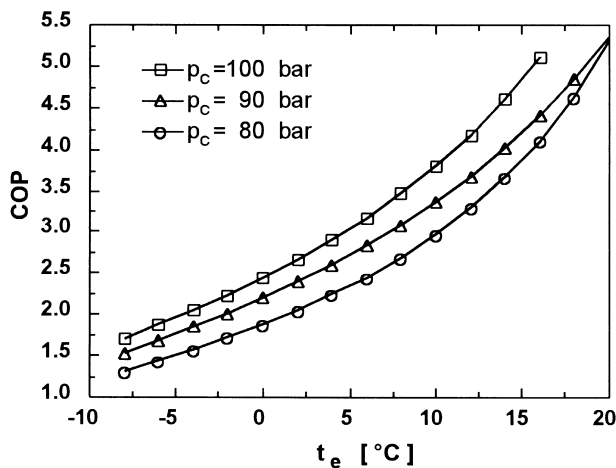


Fig. 7. The effect of the evaporation temperature  $t_e$  on the COP at different heat rejection pressures.



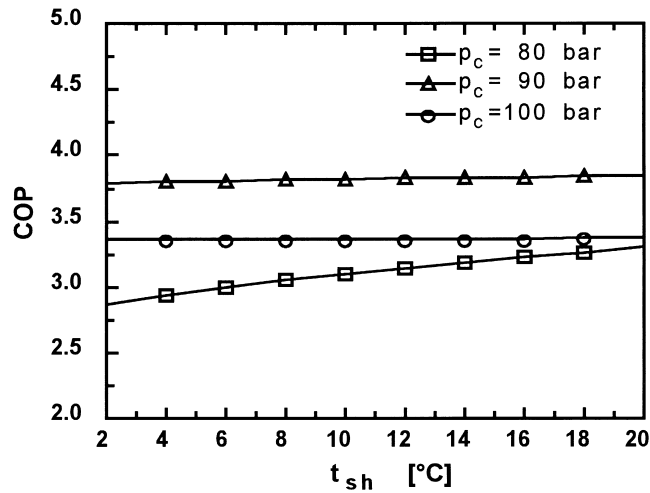


Fig. 8. The effect of the superheat  $t_{sh}$  on the COP at different heat rejection pressures.

$$p_{opt} = \frac{2.7572 + 0.1304t_e - 3.072K/C}{1 + 0.0538t_e + 0.1606K/C} t_c - \frac{8.7946 + 0.02605t_e - 105.48K/C}{1 + 0.05163t_e + 0.2212K/C} \quad (13)$$

where the temperatures are in °C while the pressures are in bar. The standard deviation between Eq. (13) and the simulated results is less than 1%.

For the special case that the isentropic efficiency  $\eta_{is}$  is a constant or independent of the heat rejection pressure  $p_c$ , it can be deduced from Eq. (9) that the optimal heat rejection pressure is a function of the evaporation temperature  $t_e$  and the outlet temperature of the gas cooler  $t_c$ , i.e.:

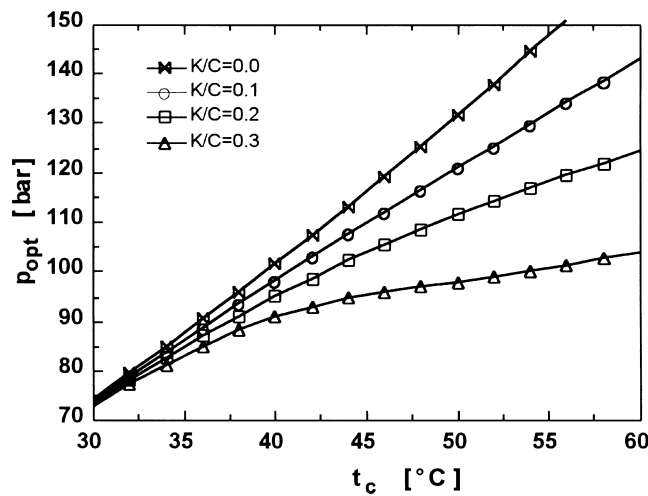


Fig. 9. The variation of the optimal heat rejection pressure with the outlet temperature of the gas cooler for different values of  $K/C$ .

$$p_{\text{opt}} = p_{\text{opt}}(t_c, t_e) \quad (14)$$

The simulating results of the optimal heat rejection pressure  $p_{\text{opt}}$  for Eq. (14) are plotted against the outlet temperature of the gas cooler for different evaporation temperature  $t_e = -5, 0, \text{ and } 10^\circ\text{C}$  in Fig. 10. The optimal heat rejection pressure is seen to increase nearly linearly with increasing the outlet temperature of the gas cooler.

A correlation for the optimal heat rejection pressure  $p_{\text{opt}}$  in terms of the evaporation temperature  $t_e$  and the outlet temperature of the gas cooler  $t_c$  based on Eq. (14) was obtained:

$$p_{\text{opt}} = (2.778 - 0.0157t_e)t_c + (0.381t_e - 9.34) \quad (15)$$

again, where the temperatures are in  $^\circ\text{C}$  while the pressures are in bar. The standard deviation between Eq. (15) and the simulated results is less than 1%.

#### 4. Concluding remarks

In this paper, we have presented a thermodynamic cycle simulation of transcritical carbon dioxide cycles. It is shown that for a transcritical carbon dioxide cycle there exists an optimal heat rejection pressure that gives a maximum COP. The analysis reveals that the values of the optimal heat rejection pressure mainly depend on the outlet temperature of the gas cooler, the evaporation temperature, and the performance of the compressor. Based on the cycle simulations, a correlation of the optimal heat rejection pressure in terms of appropriate parameters is obtained for specific conditions. The results suggest that as the heat rejection pressure varies, both the COP and the refrigeration effect of the system will change. In order to assure that the system can be operated at the optimal heat rejection pressure and meet the load requirement, a control method that simultaneously adjusts both the speed of the compressor and the opening of the throttling valve is needed.

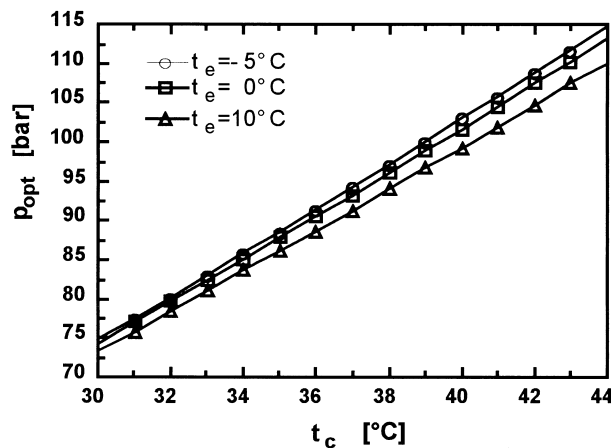


Fig. 10. The variation of the optimal heat rejection pressure with the outlet temperature of the gas cooler when  $\eta_{\text{is}} = \text{constant}$ .

## Acknowledgements

This work was supported by the National Natural Science Foundation of China under Grant 59806002 and by the Hong Kong RGC Earmarked Research Grant HKUST 6046/99E. Thanks are due to Danfoss A/S for providing the test data of CO<sub>2</sub> compressors.

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